Linear Algebra Theorems and Definitions

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All theorems, corollaries, lemmas, remarks, and asides are direct quotes from Linear Algebra, 4th Edition, by Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence

Contents

Chapter 0

List of Symbols

A_{ij}	the ij-th entry of the matrix A
A^{-1}	the inverse of the matrix A
$A^{\dagger}$	the pseudoinverse of the matrix A
A^{*}	the adjoint of the matrix A
$\tilde{A}_{ij}$	the matrix A with row i and column j deleted
A^{t}	the transpose of the matrix A
$(A\|B)$	the matrix A augmented by the matrix B
$B_1 \bigoplus \cdots \bigoplus B_k$	the direct sum of matrices B_1 through B_k
$\mathcal{B}(V)$	the set of bilinear forms on V
β^{*}	the dual basis of β
β_x	the T-cyclic basis generated by x
$\mathbb{C}$	the field of complex numbers
$\mathbb{C}_i$	the ith Gerschgorin disk
$\mathrm{cond}\,(A)$	the condition number of the matrix A
$C^n(\mathbb{R})$	set of functions f on $\mathbb{R}$ with $f^{(n)}$ continuous
C^{∞}	set of functions with derivatives of every order
$C(\mathbb{R})$	the vector space of continuous functions on $\mathbb{R}$
$C([0,1])$	the vector space of continuous functions on $[0,1]$
C_x	the T-cyclic subspaces generated by x
D	the derivative operator on C^{∞}
$\det(A)$	the determinant of the matrix A
δ_{ij}	the Kronecker delta
$\dim(V)$	the dimension of V
e^{A}	$\lim_{m\to\infty}\left(I + A + \frac{A^2}{2!} + \cdots + \frac{A^m}{m!}\right)$
e_i	the ith standard vector of $\mathbb{F}^n$

E_λ	the eigenspace of T corresponding to λ
$\mathbb{F}$	a field
$f(A)$	the polynomial $f(x)$ evaluated at the matrix A
F^n	the set of n-tuples with entries in a field $\mathbb{F}$
$f(T)$	the polynomial $f(x)$ evaluated at the operator T
$\mathcal{F}(S, \mathbb{F})$	the set of functions from S to a field $\mathbb{F}$
H	space of continuous complex functions on $[0, 2\pi]$
I_n or I	the $n \times n$ identity matrix
$\mathbb{I}_V$ or $\mathbb{I}$	the identity operator on V
K_λ	generalized eigenspace of T corresponding to λ
K_ϕ	$\{x \mid (\phi(T))^p(x) = 0$, for some positive integer $p\}$
L_A	left-multiplication transformation by matrix A
$\lim_{m\to\infty} A_m$	the limit of a sequence of matrices
$\mathcal{L}(V)$	the space of linear transformations from V to V
$\mathcal{L}(V, W)$	the space of linear transformations from V to W
$M_{m\times n}(\mathbb{F})$	the set of $m \times n$ matrices with entries in $\mathbb{F}$
$v(A)$	the column sum of the matrix A
$v_j(A)$	the jth column sum of the matrix A
$N(T)$	the null space of T
nullity (T)	the dimension of the null space of T
O	the zero matrix
per (M)	the permanent of the 2×2 matrix M
$P(\mathbb{F})$	the space of polynomials with coefficients in $\mathbb{F}$
$P_n(\mathbb{F})$	the polynomials in $P(\mathbb{F})$ of degree at most n
ϕ_β	the standard representation with respect to basis β
$\mathbb{R}$	the field of real numbers
rank (A)	the rank of the matrix A
rank (T)	the rank of the linear transformation T
$\rho(A)$	the row sum of the matrix A
$\rho_i(A)$	the ith row sum of the matrix A
$R(T)$	the range of the linear transformation T
$S_1 + S_2$	the sum of sets S_1 and S_2
span (S)	the span of the set S
$S^\perp$	the orthogonal complement of the set S
$[T]_\beta$	the matrix representation of T in basis β
$[T]_\beta^\gamma$	the matrix representation of T in bases β and γ
T^{-1}	the inverse of the linear transformation T

$T^\dagger$	the pseudoinverse of the linear transformation T
T^*	the adjoint of the linear operator T
T_0	the zero transformation
T^t	the transpose of the linear transformation T
T_θ	the rotation transformation by θ
T_W	the restriction of T to a subspace W
$\mathrm{tr}\,(A)$	the trace of the matrix A
V^*	the dual space of the vector space V
V/W	the quotient space of V modulo W
$W_1 + \cdots + W_k$	the sum of subspaces W_1 through W_k
$\sum_{i=1}^{k} W_i$	the sum of subspaces W_i through W_k
$W_1 \bigoplus W_2$	the direct sum of subspaces W_1 and W_2
$W_1 \bigoplus \cdots \bigoplus W_k$	the direct sum of subspaces W_1 through W_k
$\|x\|$	the norm of the vector $\vec{x}$
$[x]_\beta$	the coordinate vector of x relative to β
$\langle x, y \rangle$	the inner product of $\vec{x}$ and $\vec{y}$
$\mathbb{Z}_2$	the field consisting of 0 and 1
$\overline{\vec{z}}$	the complex conjugate of $\vec{z}$
$\vec{0}$	the zero vector

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